THE USE OF STANDARDIZED MATERIALS IN ARITHMETIC FOR DIAGNOSING PUPILS' METHODS OF WORK

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That there are many defective methods of working arithmetical problems involving the four fundamental operations is evident to anyone who has tabulated the results of an arithmetic test. So long as arithmetical work is carried on by means of relatively unclassified lists of problems, there is difficulty in locating with precision the particular defects that make the work of many pupils cumbersome. However, as soon as carefully selected problems are presented to a pupil, the precise nature of his difficulty is often readily discovered The purpose of this article is to set forth some of the methods found in the work of individual pupils of Grades IV to VIII and to suggest possibilities for aiding such pupils.

The method of diagnosis.—The method followed in the cases to be described was carried on in connection with the use of the Courtis practice lessons. These lessons offer an opportunity for making a careful analysis of the way in which pupils do their work in arithmetic. That is, as soon as a pupil fails to do one of the lessons in five or six days, a case for diagnosis presents itself. This pupil should be taken to a quiet part of the building and, by means of the practice lesson upon which he has been working, be diagnosed. The lessons are particularly well adapted for diagnosis, because combinations of varying difficulty are placed side by side. This makes possible a study of the child's methods while allowing him to work the problems in their regular order. One great advantage of this is that a more nearly normal response is secured than would be if skipping around were necessary; another is that pupils especially needing attention can be allowed to start across the page and work both easy and hard problems, thus showing their ability in both kinds of problems.

In actual diagnosis this last-mentioned factor is especially important. For example, if a pupil is working the fourth row of problems in the first lesson, he may do the first three problems rather easily, but the fourth and fifth problems with 9–6 combinations will present such difficulty that there will be marked retardation. If his confidence has already been secured by the person testing, queries concerning his methods of working these problems may be made. Then the following problems may be worked; the next two will probably be "easy." The ninth problem in this row has a 9–5–4 combination; this is likely to slacken the speed again. And so on through the list, one may, as a rule, question to find out the method whenever a problem is worked very much more slowly than those problems which are "easy." The character of the diagnosis will be much the same in case the pupil's breakdown has occurred with any of the other lessons.

Pupils' methods of work.—The findings as to methods employed by pupils in "difficult" combinations is both interesting and significant. The following methods were found in the work of pupils who were tried out in the manner just described. A fourth-grade boy showed by slow work that the combination 9–7–5 was difficult for him. When questioned, he showed that he used a common form of "breaking-up" the larger digits. In working the problem, he said to himself: "$9+2+2+2+1=16$ and 21." This shows that the 9–7 combination was not known, but that the 16–5 combination was, inasmuch as he arrived at "21" directly after having combined the other two numbers. Another boy of the same grade showed the same type of difficulty in a more pronounced form. He added 8, 6, and 0 as follows: "First take 4, then take 2, then add 8 and 4—makes 12, and 2 makes 14." In adding 9, 7, and 5, he said: "9 and 3 is 12 and 4 is 16 and 2—18; and 2—20; and 1—21." He broke into parts even so easy a problem as $3+4+9$, adding $9+3+2+2=16$.

A pupil from the fifth grade presented a quite different method of adding. In adding 4, 9, and 6, she explained: "Take the 6, then add 3 out of the 4. Then 9 and 9 are 18, and 1 are 19." Other problems were worked similarly: one containing 3, 9, and 8 was solved as follows: "8 and 8 are 16 and 3 are 19 and 1 are 20";

5, 6, and 9 as follows: "6, 7, 8, 9, and 9 are 18 and 2 are 20." This tendency to build up combinations of 8's or 9's continued in the case of another problem: 6, 5, and 8 were added thus: "6, 7, 8, and 8 are 16 and 3 are 19." Probably her first problem was worked similarly, but I had to have her dictate her method twice before I understood; she then gave it as quoted.

Methods which are quite as clumsy are found in the case of subtraction. One boy of the fifth grade was found to build up his subtrahend in the case of many problems. For example, in subtracting 8 from 37, he increased his subtrahend to 10, then obtained 27, and finally added 2 to 27 to compensate for the addition of 2 to 8. Likewise, in subtracting 7 from 30 he added 3 to 7 and proceeded as before. This boy knew certain combinations very well, but did problems containing other combinations by a method much harder than the correct one.

Even greater resourcefulness was shown by a fifth-grade boy who found the differences between some numbers by first dividing, then noting the remainder or lack of one, then multiplying, and finally adding to, or taking from, the result as necessary. For example, in subtracting 9 from 44, he proceeded as follows: "Nine goes into 44 five times and 1 less; 4 times 9 are 36, minus 1 equals 35." That is, this boy knew certain multiplication combinations better than he did certain subtraction processes; therefore, he used multiplication, making adjustments either upward or downward as demanded by the problem.

In addition to the use of such methods as have been described, many pupils fail to make progress merely because of lack of speed in the use of proper methods; others "count" silently; a very few count on their fingers; some have to go from certain points in the multiplication table in order to find the needed product or quotient. These methods as well as those described have at least two serious defects: (1) the danger of making errors is greater than in the use of simpler methods, owing to the larger number of operations involved; (2) for the same reason, the speed of working is greatly reduced.

Remedying the defects.—In making use of the results of diagnoses, it is often found to be valuable to the pupil to show him that

another method is more effective than the one he is using. This can be done by timing him while he works several problems in his own way and then drilling him in the use of a correct method upon the same problems or different ones, finally returning to the problems first worked and timing him again. There is practically no danger of failing to secure an appreciable gain in time. Interest in the lessons is sufficient to insure at least an attempt upon the part of the pupil to use the better method. Further individual drill is, of course, the kind of training that is needed.

The practice lessons themselves should be used in the individual drill work. Mere repetition, supplemented by attempts to beat one's own record, usually results in automatizing the troublesome combinations in from three to five days. The drill periods need to be in the early hours of the day and should be brief and snappy; few of the pupils needing drill profit by a period longer than ten minutes. Occasionally, a pupil is unable, even with drill, to do certain of the lessons; also, some pupils are unable to write the answers rapidly, but are able to give them orally; as there are few of such cases, those pupils should be advanced to succeeding lessons.

Many more methods by which pupils work problems might be found; enough have been cited to show the presence of subterfuges which are employed to conceal lack of knowledge of, and ability to use readily, the proper methods. That individual attention is needed in case of many slow pupils is not open to question; only through such attention is it possible to treat the specific difficulties of these pupils. That such attention will produce the desired result has been shown in the case of scores of children who have been drilled by the writer. Marked improvement is always shown in the ability to work the standardized lessons and to make better scores in standardized tests. The other arithmetical work of pupils is usually reported upon as improving, although the immediate effect of specialized drill is not always apparent; it is often found, however, that the simple lessons are the means of locating a difficulty that has retarded seriously a pupil's general progress. Such lessons show just how far a pupil *can* go and also just the kind of drill that *he* needs. They enable the teacher to deal directly with the precise form of the mechanics of arithmetical work that stands in need of improvement.